Centre Number | 1 | 2 | 5 |

CRANBROOK SCHOOL

Student Number | | | | | | | | |

2017

Trial HSC

Assessment Task 4

Mathematics

Reading time	5 minutes
Writing time	3 hours
Total Marks	100
Task weighting	35%

General Instructions

- Write using blue or black pen
- Diagrams drawn using pencil
- A Board-approved calculator may be used
- The Reference sheet is on the last page of this booklet
- Use the Multiple-Choice Answer Sheet provided
- All relevant working should be shown for each question

Additional Materials Needed

- Multiple Choice Answer Sheet
- 6 writing booklets

Structure & Suggested Time Spent

Section I

Multiple Choice Questions

- Answer Q1 – 10 on the multiple choice answer sheet
- Allow 20 minutes for this section

Section II

Extended response Questions

- Attempt all questions in this section in a separate writing booklet
- Allow about 160 minutes for this section

This paper must not be removed from the examination room

Disclaimer

The content and format of this paper does not necessarily reflect the content and format of the HSC examination paper.

Section I

10 Marks

Allow about 20 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

---

1 What is 25.09582 correct to 4 significant figures?

(A) 25.09

(B) 25.10

(C) 25.095

(D) 25.096

2 The quadratic equation $x^2 + 3x - 1 = 0$ has roots α and β. What is the value of

$\alpha\beta + (\alpha + \beta)$?

(A) 4

(B) 2

(C) –4

(D) –2

3 The semi-circle $y = \sqrt{9 - x^2}$ is rotated about the x-axis. Which of the following expressions is correct for the volume of the solid of revolution?

(A) $V = \pi \int_0^3 \left(9 - x^2\right) dx$

(B) $V = 2\pi \int_0^3 \left(9 - x^2\right) dx$

(C) $V = \pi \int_0^3 \left(9 - y^2\right) dy$

(D) $V = 2\pi \int_0^3 \left(9 - y^2\right) dy$

4 The curve $y = 2x^3 + ax^2 - 3$ has a point of inflexion at $x = 1$. The value of a is:

(A) 6

(B) $\frac{3}{2}$

(C) 0

(D) -6

5 What is the derivative of $(1+\log_e x)^4$?

(A) $4(1+\log_e x)^3$

(B) $\dfrac{(1+\log_e x)^5}{5}$

(C) $\dfrac{4(1+\log_e x)^3}{x}$

(D) $\dfrac{(1+\log_e x)^5}{5x}$

6 The area between the curve $y=\dfrac{1}{x}$, the x-axis and the lines $x=1$ and $x=b$ is equal to 2 square units. The value of b is:

(A) e

(B) e^2

(C) $2e$

(D) 3

7 For which values of x is the curve $f(x) = 2x^3 + x^2$ concave down?

(A) $x < -\frac{1}{6}$

(B) $x > -\frac{1}{6}$

(C) $x < -6$

(D) $x > 6$

8 What is the angle of inclination of the line $3x + 2y = 7$ with the positive direction of the x – axis?

(A) $33^\circ 41'$

(B) $56^\circ 19'$

(C) $123^\circ 41'$

(D) $146^\circ 19'$

9 Solve $\sin x = \frac{\sqrt{3}}{2}$ for $0^\circ \le x \le 360^\circ$

(A) 60° or 240°

(B) 30° or 150°

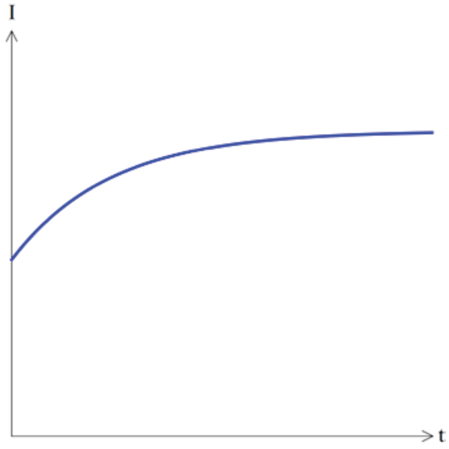
(C) 30° or 210°

(D) 60° or 120°

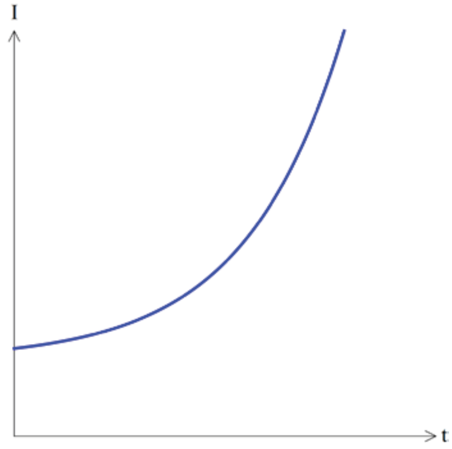
10 Interest rates are increasing at a decreasing rate.

Which of the following graphs represents the above statement?

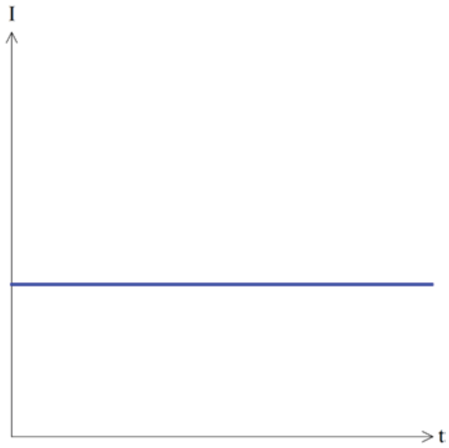
(A)



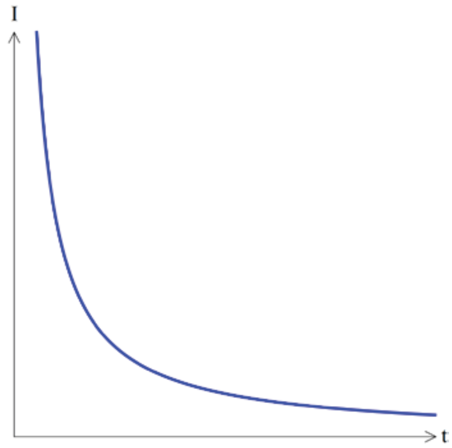
(B)



(C)



(D)



END OF SECTION I

Section II

90 Marks

Allow about 160 minutes for this section

Answer question 11–16 in separate booklets.

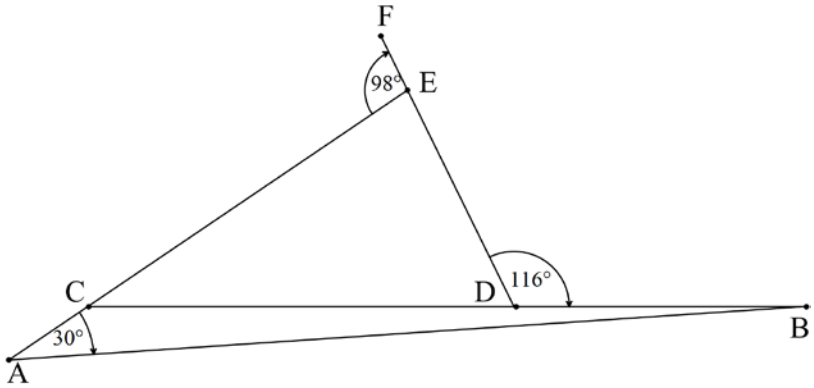
Question 11 **Start a new booklet** **15 Marks**

(a) Simplify $6-2(4-2p)$ **1**

(b) Find the derivative of $x\log_e x^2$ **2**

(c) Given that a and b are integers find the values of a and b if $(5-3\sqrt{2})^2 = a-\sqrt{b}$. **3**

(d)



In the diagram $\angle CAB = 30^o$, $\angle EDB = 116^o$, and $\angle AEF = 98^o$.

Find the size of $\angle ABC$. **2**

Question 11 continues on the next page.

(e) Solve for $0 \le x \le 2\pi$, $2\cos^2 x = 1$. **2**

(f) Differentiate $\dfrac{3x}{2x^2 - 1}$ with respect to x leaving your answer in simplest form. **2**

(g) Find $\int_0^{\frac{\pi}{9}} 4\sec^2(3x)\,dx$ leaving your answer in exact form. **3**

END OF QUESTION 11

Question 12 **Start a new booklet** **15 Marks**

(a) For the parabola $(x-2)^2 = 4y$

(i) Find the coordinates of the vertex **1**

(ii) State the equation of the directrix of the parabola **1**

(b) Consider the quadratic equation $2x^2 + 4x - k = 0$

(i) Write down the discriminant of this equation. **1**

(ii) For what values of k does $2x^2 + 4x - k = 0$ have real roots. **1**

(c) The velocity of a particle moving along the x axis is given by **2**

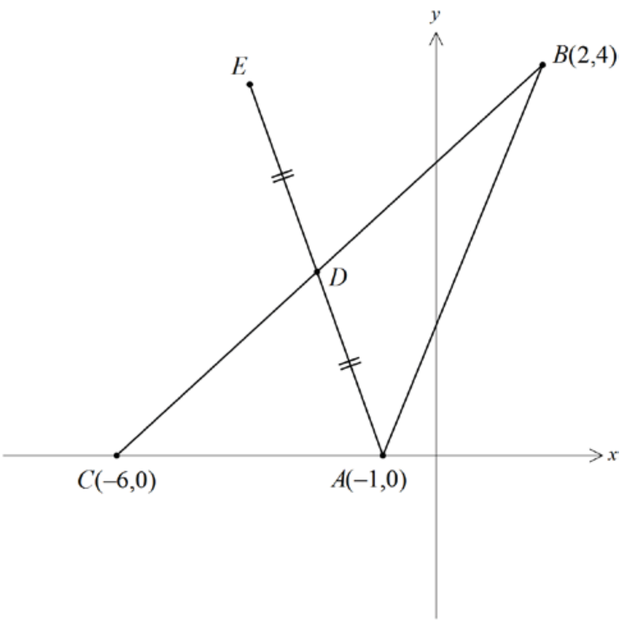
$v = b + \dfrac{c}{t+1}$, where b and c are constant.

Given that the particle has initial velocity 2 ms^{-1} and its initial acceleration was 4 ms^{-2}, find the values of b and c.

Question 12 continues on the next page.

(d) In the diagram A, B, C and D are the points $(-1,0)$, $(2,4)$, $(-6,0)$ and $(-2,2)$ respectively.

D is also the midpoint of AE.



(i) Find the length of the interval AB. **1**

(ii) Find the equation of the circle with centre at B which passes through the point A. **1**

(iii) Show that the size of $\angle CAB$ is 127° to the nearest degree. **1**

(iv) Find the midpoint of BC. **1**

(v) Show that the equation of the line BC is $x - 2y + 6 = 0$ **1**

(vi) Find the perpendicular distance of A from the line BC in simplest exact form. **2**

(vii) What type of quadrilateral is $ABEC$? Give reasons for your answer. **2**

END OF QUESTION 12

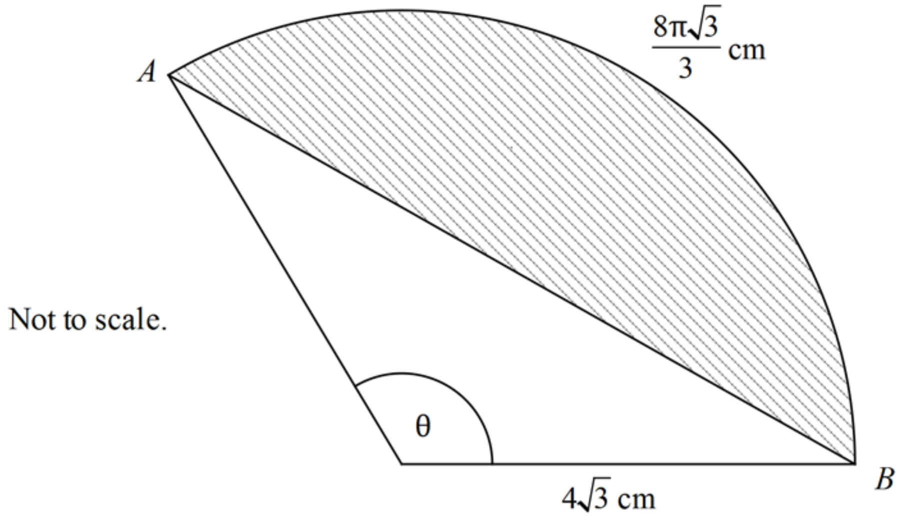
Question 13 **Start a new booklet** **15 Marks**

(a) Use Simpson's Rule, with 3 function values, to find the area between the curve $y = xe^{\sqrt{x}}$, the x axis, and the lines $x = 4$ and $x = 9$. **4**

Give your answer correct to 2 decimal places.

(b) The diagram shows a sector with angle θ at the centre and radius $4\sqrt{3}$ cm.

The arc length is $\dfrac{8\pi\sqrt{3}}{3}$ cm.



(i) Find the size of angle θ. **1**

(ii) Find the length of chord AB. **2**

(iii) Find the exact area of the shaded minor segment. Leave your answer in its simplest form. **2**

Question 13 continues on the next page.

(c) Anthony bought a second hand car. Its odometer read 10 500 km on the day he bought it.

He drove the car for 250 km in the first week, 270 km in the second week and in each successive week he drove it 20 km more than the previous week.

(i) What distance did he drive in the 15^{th} week? **2**

(ii) What distance will he drive this car during the first 15 weeks? **2**

(iii) In how many weeks will his car's odometer show 20 620 km? **2**

END OF QUESTION 13

Question 14 **Start a new booklet** **15 Marks**

(a) The sum of the first 2 terms of a geometric progression is $\frac{8}{9}$ of its limiting sum. **2**

Find the common ratio.

(b) A particle moves in a straight line so that its displacement, x metres from a fixed point on a line is given by $x = t + \frac{25}{t+2}$, where t is measured in seconds.

(i) Find the particle's initial position. **1**

(ii) Find expressions for the velocity and acceleration in terms of t. **2**

(iii) Find when and where the particle is at rest. **2**

(iv) What value does the velocity approach as $t \to \infty$? **1**

(v) Explain why the particle is never to left of its initial position. **1**

Question 14 continues on the next page.

(c) The number N of bacteria in a culture at a time t seconds is given by the equation

$N = Ae^{kt}$.

(i) If the initial number of bacteria is 25 000, and after 10 seconds there are 26 813 bacteria, find the values of A and k. **2**

(ii) Determine the number of bacteria after 30 seconds (to the nearest whole number). **1**

(iii) After what time period will the number of bacteria have doubled? **2**

(iv) At what rate is the number of bacteria increasing when $t = 30$ seconds? **1**

END OF QUESTION 14

Question 15 **Start a new booklet** **15 Marks**

(a) A function is defined by $f(x) = x^3 - 3x^2 - 9x + 22$.

(i) Find the coordinates of the turning points of the graph $y = f(x)$ and determine their nature. **4**

(ii) Find the coordinates of the point(s) of inflexion. **2**

(iii) Hence, sketch the graph of $y = f(x)$, showing the turning points, the point(s) of inflexion and the *y*-intercept. **3**

(b) Consider the curve $y = 4x^2(1 - x)$.

(i) Show that the gradient of the tangent at the point (1, 0) is –4. **1**

(ii) Find the equations of the tangent and normal to this curve at the point (1,0) **3**

(iii) The tangent and normal cut the *y*-axis at *A* and *B* respectively. **2**

If the point of intersection of the tangent and the normal is *C*, find the area of ΔABC.

END OF QUESTION 15

Question 16 **Start a new booklet** **15 Marks**

(a) Robert borrows \$400 000 to buy a house. The interest rate is 6% p.a. compounding monthly. He agrees to repay the loan in 30 years with equal monthly repayments of \$$M$. Let \$$A_n$ be the amount owing after the n^{th} repayment.

(i) If the amount owing after two repayments A_2 is \$399 201.61, show that his monthly repayment is \$$M$ = \$2 398.20. **2**

(ii) Show that $A_n = \$479\,640 - \$79\,640 \times 1.005^n$. **2**

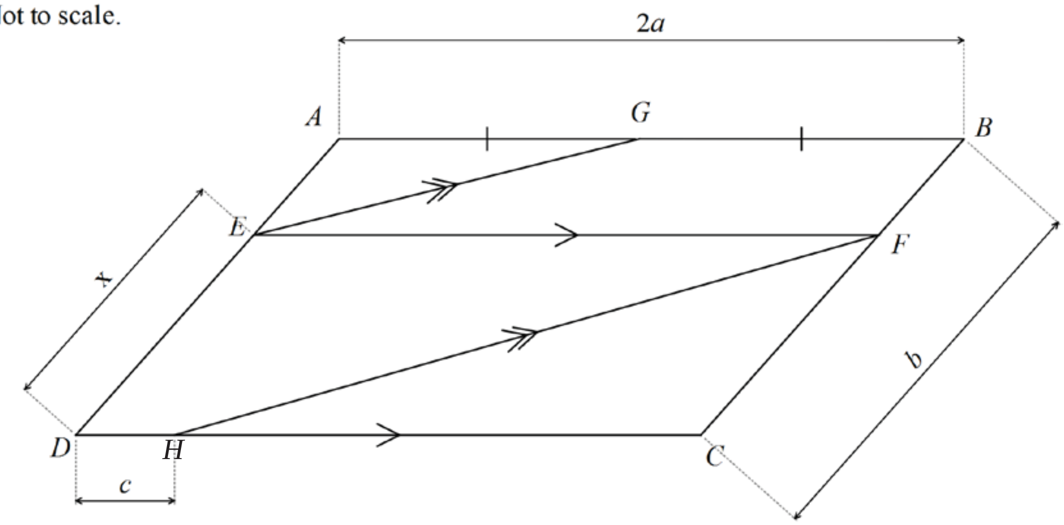
(iii) After how many months will the amount owing be less than \$150 000? **2**

Question 16 continues on the next page.

(b) $ABCD$ is a parallelogram with sides $AB = 2a$ and $BC = b$. EF is parallel to AB and DC, and $DE = x$.

G is the midpoint of AB. The line parallel to EG from F intersects CD at H where $DH = c$.

Not to scale.



(i) Show that ΔAEG is similar to ΔCFH. **2**

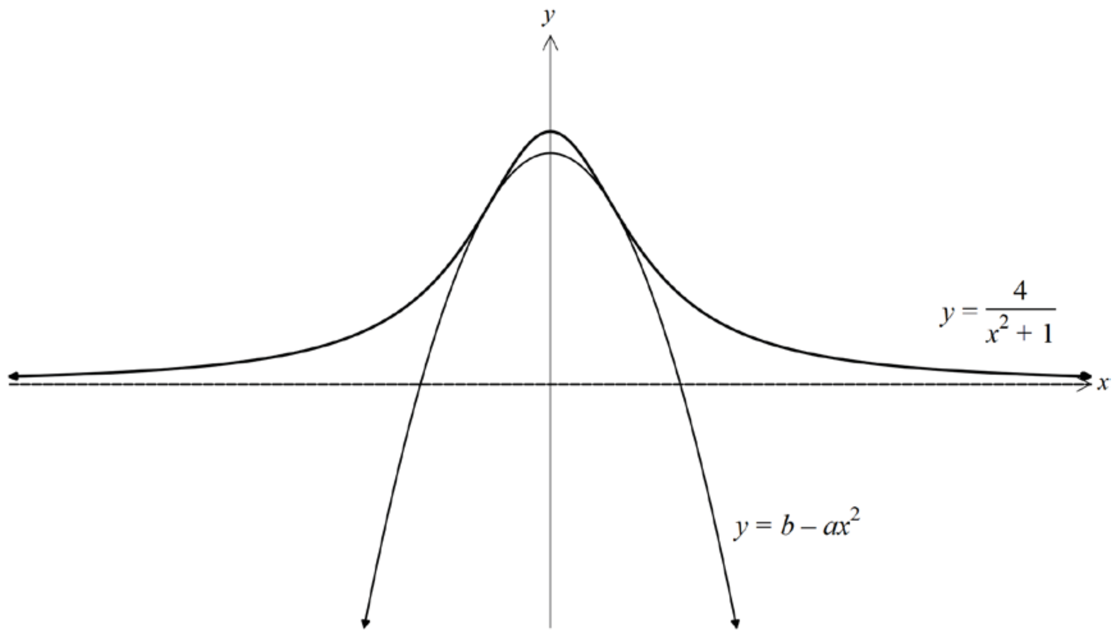
(ii) Find an expression for x in terms of a, b and c. **3**

Question 16 continues on the next page.

(c) The parabola $y = b - ax^2$, where $a > 0$ and $b > 0$, is under the curve $y = \dfrac{4}{x^2+1}$.

The parabola touches the curve at two points that are symmetrical at the y-axis, as shown in the diagram below.

Let the two curves intersect at $x = \pm q$.



(i) Show $aq^4 + (a-b)q^2 + 4 - b = 0$. **1**

(ii) Hence show that $b = 4\sqrt{a} - a$. **2**

(iii) Hence show $0 < a < 4$. **1**

END OF EXAM

BOSTES | BOARD OF STUDIES TEACHING & EDUCATIONAL STANDARDS NSW

2016 HIGHER SCHOOL CERTIFICATE EXAMINATION

REFERENCE SHEET

– Mathematics –

– Mathematics Extension 1 –

– Mathematics Extension 2 –

Mathematics

Factorisation

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Angle sum of a polygon

$$S = (n-2) \times 180°$$

Equation of a circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Trigonometric ratios and identities

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

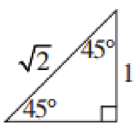
$$\sec\theta = \frac{1}{\cos\theta}$$

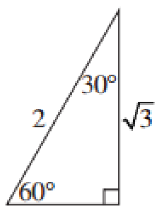
$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$

Exact ratios





Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of a triangle

$$\text{Area} = \frac{1}{2}ab\sin C$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Perpendicular distance of a point from a line

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Point-gradient form of the equation of a line

$$y - y_1 = m(x - x_1)$$

***n*th term of an arithmetic series**

$$T_n = a + (n-1)d$$

Sum to *n* terms of an arithmetic series

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{or} \quad S_n = \frac{n}{2}(a + l)$$

***n*th term of a geometric series**

$$T_n = ar^{n-1}$$

Sum to *n* terms of a geometric series

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad \text{or} \quad S_n = \frac{a(1 - r^n)}{1 - r}$$

Limiting sum of a geometric series

$$S = \frac{a}{1-r}$$

Compound interest

$$A_n = P\left(1 + \frac{r}{100}\right)^n$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives

If $y = x^n$, then $\dfrac{dy}{dx} = nx^{n-1}$

If $y = uv$, then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v}$, then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

If $y = F(u)$, then $\dfrac{dy}{dx} = F'(u)\dfrac{du}{dx}$

If $y = e^{f(x)}$, then $\dfrac{dy}{dx} = f'(x)e^{f(x)}$

If $y = \log_e f(x) = \ln f(x)$, then $\dfrac{dy}{dx} = \dfrac{f'(x)}{f(x)}$

If $y = \sin f(x)$, then $\dfrac{dy}{dx} = f'(x)\cos f(x)$

If $y = \cos f(x)$, then $\dfrac{dy}{dx} = -f'(x)\sin f(x)$

If $y = \tan f(x)$, then $\dfrac{dy}{dx} = f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$\alpha + \beta = -\dfrac{b}{a}$ $\qquad$ $\alpha\beta = \dfrac{c}{a}$

Equation of a parabola

$$(x-h)^2 = \pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$\int e^{ax+b} \, dx = \frac{1}{a}e^{ax+b} + C$$

$$\int \frac{f'(x)}{f(x)} \, dx = \ln|f(x)| + C$$

$$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$$

$$\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$$

$$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{2}\left[f(a) + f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)\,dx \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

Logarithms – change of base

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Angle measure

$180° = \pi$ radians

Length of an arc

$l = r\theta$

Area of a sector

$\text{Area} = \dfrac{1}{2}r^2\theta$

Mathematics Extension 1

Angle sum identities

$\sin(\theta+\phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$

$\cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi$

$\tan(\theta+\phi) = \dfrac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$

***t* formulae**

If $t = \tan\dfrac{\theta}{2}$, then

$$\sin\theta = \frac{2t}{1+t^2}$$

$$\cos\theta = \frac{1-t^2}{1+t^2}$$

$$\tan\theta = \frac{2t}{1-t^2}$$

General solution of trigonometric equations

$\sin\theta = a, \qquad \theta = n\pi + (-1)^n \sin^{-1} a$

$\cos\theta = a, \qquad \theta = 2n\pi \pm \cos^{-1} a$

$\tan\theta = a, \qquad \theta = n\pi + \tan^{-1} a$

Division of an interval in a given ratio

$$\left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$$

Parametric representation of a parabola

For $x^2 = 4ay$,

$$x = 2at, \quad y = at^2$$

At $\left(2at, at^2\right)$,

tangent: $y = tx - at^2$

normal: $x + ty = at^3 + 2at$

At $\left(x_1, y_1\right)$,

tangent: $xx_1 = 2a\left(y + y_1\right)$

normal: $y - y_1 = -\dfrac{2a}{x_1}\left(x - x_1\right)$

Chord of contact from $\left(x_0, y_0\right)$: $xx_0 = 2a\left(y + y_0\right)$

Acceleration

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

Simple harmonic motion

$x = b + a\cos(nt + \alpha)$

$\ddot{x} = -n^2(x - b)$

Further integrals

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a} + C$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

Sum and product of roots of a cubic equation

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{d}{a}$$

Estimation of roots of a polynomial equation

Newton's method

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

Binomial theorem

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k} = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

Multiple Choice, Page 1

Wednesday, 12 July 2017 11:11 AM

Q1

$\underbrace{25.09}582$ Round up

$= 25.10 \quad \therefore$ Answer B

Q2

$x^2 + 3x - 1 = 0$

$\alpha + \beta = -3$

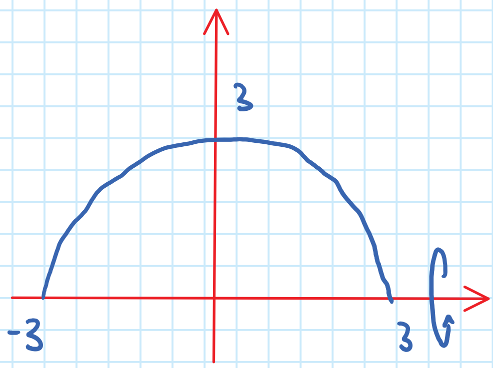
$\alpha\beta = -1$

$\therefore \quad \alpha\beta + (\alpha+\beta)$

$= -1 - 3$

$= -4 \quad \therefore$ Answer C

Q3



$V = \pi \int_{-3}^{3} y^2 \, dx$

$= \pi \int_{-3}^{3} 9 - x^2 \, dx$

or $= 2\pi \int_{0}^{3} 9 - x^2 \, dx$

$\therefore$ Answer B

Multiple Choice, Page 2

Wednesday, 12 July 2017 11:11 AM

Q4

$y = 2x^3 + ax^2 - 3$ POI @ $x = 1$

$\frac{dy}{dx} = 6x^2 + 2ax$

$\frac{d^2y}{dx^2} = 12x + 2a$

@ $x = 1$, $\frac{d^2y}{dx^2} = 0$

$\therefore 0 = 12 + 2a$

$a = -6$ Answer D

Q5

$y = (1 + \ln x)^4$

$\frac{dy}{dx} = 4(1 + \ln x)^3 \times \frac{1}{x}$ Answer C

Q6

$A = \int_1^b \frac{1}{x}\,dx$

$2 = \left[\ln x\right]_1^b$

$2 = \ln b$

$b = e^2$ Answer B

Multiple Choice, Page 3

Wednesday, 12 July 2017   11:11 AM

Q7 Concave down, $\frac{d^2y}{dx^2} < 0$

$$f''(x) < 0$$

$$f'(x) = 6x^2 + 2x$$

$$f''(x) = 12x + 2$$

$$12x + 2 < 0$$

$$12x < -2$$

$$x < -\frac{1}{6}$$

Answer A

Q8 $3x + 2y = 7$

$$y = -\frac{3}{2}x + \frac{7}{2}$$

$$\therefore \quad m = -\frac{3}{2}$$

$$\theta = \tan^{-1}\left(-\frac{3}{2}\right)$$

$$= 123^\circ 41'$$

Answer C

Q9 $\sin x = \frac{\sqrt{3}}{2}$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ$$

$$\therefore \quad x = 60^\circ, \quad 180 - 60^\circ$$

$$= 60^\circ, \ 120^\circ$$

$\therefore$ Answer D

Wednesday, 12 July 2017 11:11 AM

Q10 Increasing at a decreasing rate

gradients are positive

gradients getting smaller

∴ Answer A

Multiple Choice Answers

1	2	3	4	5	6	7	8	9	10
B	C	B	D	C	B	A	C	D	A

Question 11, Page 1

Wednesday, 12 July 2017 11:16 AM

(a) $6 - 2(4 - 2p)$

$= 6 - 8 + 4p$

$= 4p - 2$ ①

(b) $\frac{d}{dx}\, x \ln x^2$

$u = x$ $\quad v = \ln x$

$\frac{du}{dx} = 1$ $\quad \frac{dv}{dx} = \frac{2}{x}$

$= x \times \frac{2}{x} + 1 \times \ln x$

① Product rule

$= 2 + \ln x^2$ ①

(c) $(5 - 3\sqrt{2})^2 = a - \sqrt{b}$

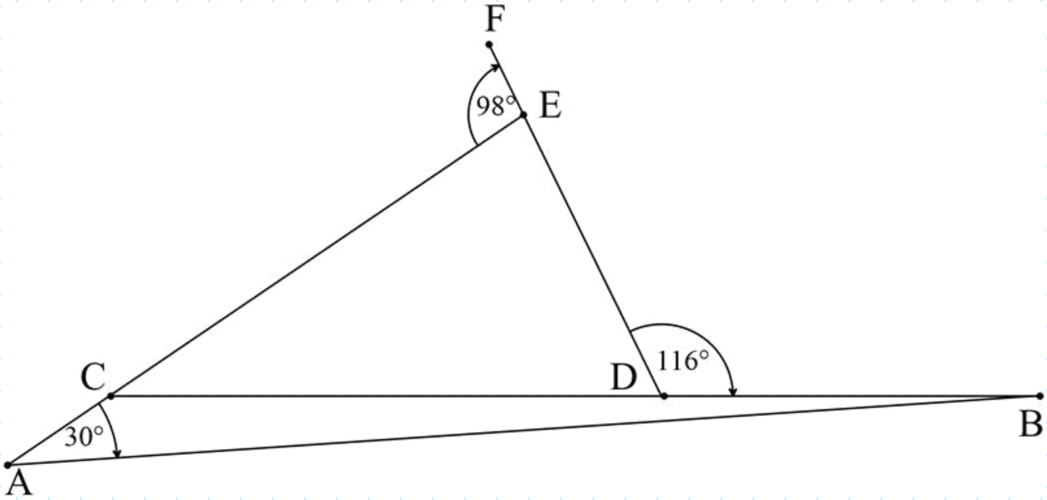
LHS $= 25 - 30\sqrt{2} + 9 \times 2$ ①

$= 43 - \sqrt{900 \times 2}$

$= 43 - \sqrt{1800}$ ①

$\therefore \quad a = 43, \quad b = 1800$ ① Must state values of a & b

(d)



$\angle FEC = 82°$ (Supplementary $\angle$'s) 1

$\angle ECD = 116 - 82$ (exterior $\angle$ of ΔECD is equal to the sum of the interior opposites)

$= 34°$ 1

$\therefore \angle ABC = 34 - 30$ (exterior $\angle$ of ΔABC is equal to the sum of interior opposites)

$= 4°$

(e) $2\cos^2 x = 1 \qquad 0 \le x \le 2\pi$

$\cos x = \pm \frac{1}{\sqrt{2}}$ $\therefore$ All 4 quadrants

$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ 1

$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ 1

(f) let $y = \dfrac{3x}{2x^2-1}$ $\begin{matrix} u \\ v \end{matrix}$

$u = 3x$ $\quad$ $v = 2x^2 - 1$

$\dfrac{du}{dx} = 3$ $\quad$ $\dfrac{dv}{dx} = 4x$

$$\frac{dy}{dx} = \frac{3(2x^2-1) - 3x \times 4x}{(2x^2-1)^2} \quad 1$$

$$= \frac{6x^2 - 3 - 12x^2}{(2x^2-1)^2}$$

$$= \frac{-3(1 + 2x^2)}{(2x^2-1)^2} \quad 1$$

(g) $4\int_0^{\frac{\pi}{9}} \sec^2 3x \; dx$ $\quad$ $\dfrac{d}{dx} \tan x = \sec^2 x$

$$\therefore = 4\left[\frac{\tan 3x}{3}\right]_0^{\frac{\pi}{9}} \quad 1$$

$$= \frac{4}{3}\left[\tan\frac{\pi}{3} - \tan 0\right] \quad 1$$

$$= \frac{4\sqrt{3}}{3} \quad 1$$

(a) (i) $(x-2)^2 = 4y$ $(x-h)^2 = 4a(y-k)$

$\therefore$ vertex @ $(2,0)$ 1

focal length $= 1$

concave up

(ii) concave up $\therefore$ directrix one focal length below the vertex

$y = -1$

(b) $2x^2 + 4x - k = 0$

(i) $\Delta = b^2 - 4ac$

$= 16 - 4 \times 2 \times -k$

$= 16 + 8k$ 1

(ii) Real roots means $\Delta \geq 0$

$\therefore 16 + 8k \geq 0$

$8k \geq -16$

$k \geq -2$ 1 Also accept $k > -2$

(c) $v = b + \dfrac{c}{t+1}$

When $t=0$ $\quad v=2$, $a=4$

$a = \dfrac{dv}{dt}$

$\dfrac{dv}{dt} = \dfrac{-c}{(t+1)^2}$ (1)

Also $2 = b + c$

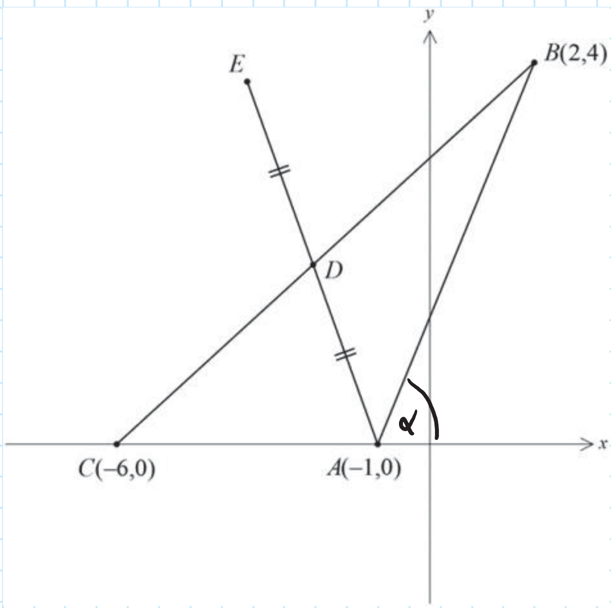
$2 = b - 4$

$b = 6$

$\therefore\ 4 = \dfrac{-c}{1^2}$

$c = -4$ (1)

(d)



(i) $AB = \sqrt{(2+1)^2 + (4-0)^2}$

$= \sqrt{9+16}$

$= 5$ (1)

(ii) centre $(2,4)$

radius $= 5$

$\therefore\ (x-2)^2 + (y-4)^2 = 25$ (1)

(iii) $m_{AB} = \dfrac{4-0}{2+1}$

$= \dfrac{4}{3}$

$\therefore\ \tan\alpha = \dfrac{4}{3}$ (1)

$\angle CAB = 180^\circ - \tan^{-1}\left(\dfrac{4}{3}\right)$

$= 127^\circ$ (Nearest degree)

(iv) Midpoint (x_m, y_m)

$$x_m = \frac{-6+2}{2} = -2 \qquad y_m = \frac{0+4}{2} = 2$$

$\therefore (-2, 2)$ ①

(v) $$m_{BC} = \frac{4-0}{2+6} = \frac{1}{2}$$

$$\therefore \frac{1}{2} = \frac{y-0}{x+6} \quad ①$$

$$x+6 = 2y$$

$$x - 2y + 6 = 0$$

(vi) $$D = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$$

$$= \frac{|(1\times -1) - 2\times 0 + 6|}{\sqrt{1^2+2^2}} \quad ①$$

$$= \frac{5}{\sqrt{5}}$$

$$= \sqrt{5} \quad ①$$

(vii) ABEC is a Rhombus (1) as
- diagonals bisect each other
- Adjacent sides are equal (1)

$$\begin{array}{c|ccc} & \overset{h}{\downarrow} & \overset{h}{\downarrow} & \downarrow \\ x & 4 & 6.5 & 9 \\ y & 4e^2 & 6.5e^{\sqrt{6.5}} & 9e^3 \end{array}$$

(a) $A \doteqdot \frac{h}{3}\left(y_0 + 4y_1 + y_2\right)$

$A \doteqdot \frac{2.5}{3}\left(4e^2 + 4\times 6.5e^{\sqrt{6.5}} + 9e^3\right)$

$\doteqdot 452.62$ (2dp)

1 $h = 2.5$

1 y coordinates

1 Correct area

1 Rounding

(b) (i) Arc length $= r\theta$

$\frac{8\pi\sqrt{3}}{3} = 4\sqrt{3}\,\theta$

$\theta = \frac{2\pi}{3}$

(ii) By the cosine rule

$AB^2 = (4\sqrt{3})^2 + (4\sqrt{3})^2 - 2\times 4\sqrt{3}\times 4\sqrt{3}\times\cos\frac{2\pi}{3}$ 1

$= 2(48) - 2\times 48\times\left(-\frac{1}{2}\right)$

$= 2(48) + 48$

$AB^2 = 144$

$AB = 12$ cm 1

(iii) $A_{segment} = A_{sector} - A_{TRIANGLE}$

$$= \frac{(4\sqrt{3})^2}{2} \times \frac{2\pi}{3} - \frac{(4\sqrt{3})^2}{2} \sin\frac{2\pi}{3} \quad 1$$

$$= \frac{(4\sqrt{3})^2}{2}\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

$$= 4\left(4\pi - 3\sqrt{3}\right) \text{ cm}^2 \quad 1$$

(c) (i) Distance travelled per week is an arithmetic progression

$a = 250$, $d = 20$ 1

$\therefore T_n = 250 + (n-1)\,20$

$T_{15} = 250 + 14 \times 20$

$= 530$ km 1

(ii) $S_{15} = \frac{15}{2}\left(2 \times 250 + (15-1) \times 20\right)$ 1

$= 5850$ km 1

(iii) Started at 10,500 km

$\therefore$ How many weeks to add 10120 km

$\therefore S_n = 10120$ 1

$$\therefore \frac{n}{2}\left(2\times 250 + (n-1)\times 20\right) = 10120$$

$$500n + 20n^2 - 20n = 20240$$

$$20n^2 + 480n - 20240 = 0$$

$$n^2 + 24n - 1012 = 0$$

$$n = \frac{-24 \pm \sqrt{576 + 4\times 1\times 1012}}{2}$$

$$= 22 \ \& \ -46$$

$n > 0$

$\therefore \quad n = 22$

$\therefore$ It will take 22 weeks. 1

(a) $S_\infty = \dfrac{a}{1-r}$ $T_1 = a$

$T_2 = ar$

$\therefore \quad a + ar = \dfrac{8}{9}\left(\dfrac{a}{1-r}\right)$ 1

$\cancel{a}(1+r) = \dfrac{8\cancel{a}}{9(1-r)}$

$(1+r)(1-r) = \dfrac{8}{9}$

$1 - r^2 = \dfrac{8}{9}$

$r^2 = \dfrac{1}{9}$

$r = \pm\dfrac{1}{3}$

(b) $x = t + \dfrac{25}{t+2}$

(i) $t = 0, \quad x = 25/2$ metres

(ii) $v = \dfrac{dx}{dt}$

$= 1 - 25(t+2)^{-2}$

$= 1 - \dfrac{25}{(t+2)^2}$ 1

$a = \dfrac{dv}{dt}$

$= \dfrac{50}{(t+2)^3}$ 1

(iii) Find when $v = 0$

$$0 = 1 - \frac{25}{(t+2)^2}$$

$$(t+2)^2 = 25$$

$$t+2 = \pm 5$$

$$t = -7 \text{ or } 3$$

As $t > 0$, $t = 3$ 1

$x = 8$ 1

(iv) As $t \to \infty$ $\frac{1}{t} \to 0$ (As the denominator gets very large)

$\therefore \frac{25}{(t+2)^2} \to 0$

$\therefore v \to 1$ 1

(v) As $t \to 0$

$$x = t + \frac{25}{t+2}$$

Always > 0 Always > 0 1

$\therefore x$ is always positive

(c) $N = Ae^{kt}$

(i) $t = 0 \quad N = 25{,}000 \ldots ①$

$t = 10 \quad N = 26813 \ldots ②$

① $25000 = A \times e^{0}$

$\therefore A = 25000$ (1)

② $26813 = 25{,}000\, e^{10k}$

$e^{10k} = \dfrac{26813}{25000}$

$10k = \ln \dfrac{26813}{25000}$

$k = \dfrac{\ln\left(\frac{26813}{25000}\right)}{10}$

$= 0.0070011196 \ldots$ (1)

(ii) $t = 30$

$N = Ae^{30k}$

$= 30843$ (1) (Nearest whole number)

(iii) $N = Ae^{kt}$

$\therefore e^{kt} = 2$

$t = \dfrac{\ln 2}{k}$ (1)

$= 99.0054\ldots$

$= 99$ seconds (nearest second) (1)

(iv) $\dfrac{dN}{dt} = kAe^{kt} = kN$

When $t = 30$

$\dfrac{dN}{dt} = k \times Ae^{30k}$

$= 216$ bacteria/second (nearest whole number)

(1)

(a) (i) $f(x) = x^3 - 3x^2 - 9x + 22$

$f'(x) = 3x^2 - 6x - 9$ 1

$f''(x) = 6x - 6$

Turning points occur when $f'(x) = 0$

$0 = 3x^2 - 6x - 9$

$= 3(x^2 - 2x - 3)$

$= 3(x-3)(x+1)$

$x = 3$ 1 $y = -5$ 1 $f''(x) = 12 \therefore$ Min

1 TEST

$x = -1$ $y = 27$ $f''(x) = -12 \therefore$ Max

(ii) Points of Inflexion occur when $f''(x) = 0$

$6x - 6 = 0$

$x = 1$

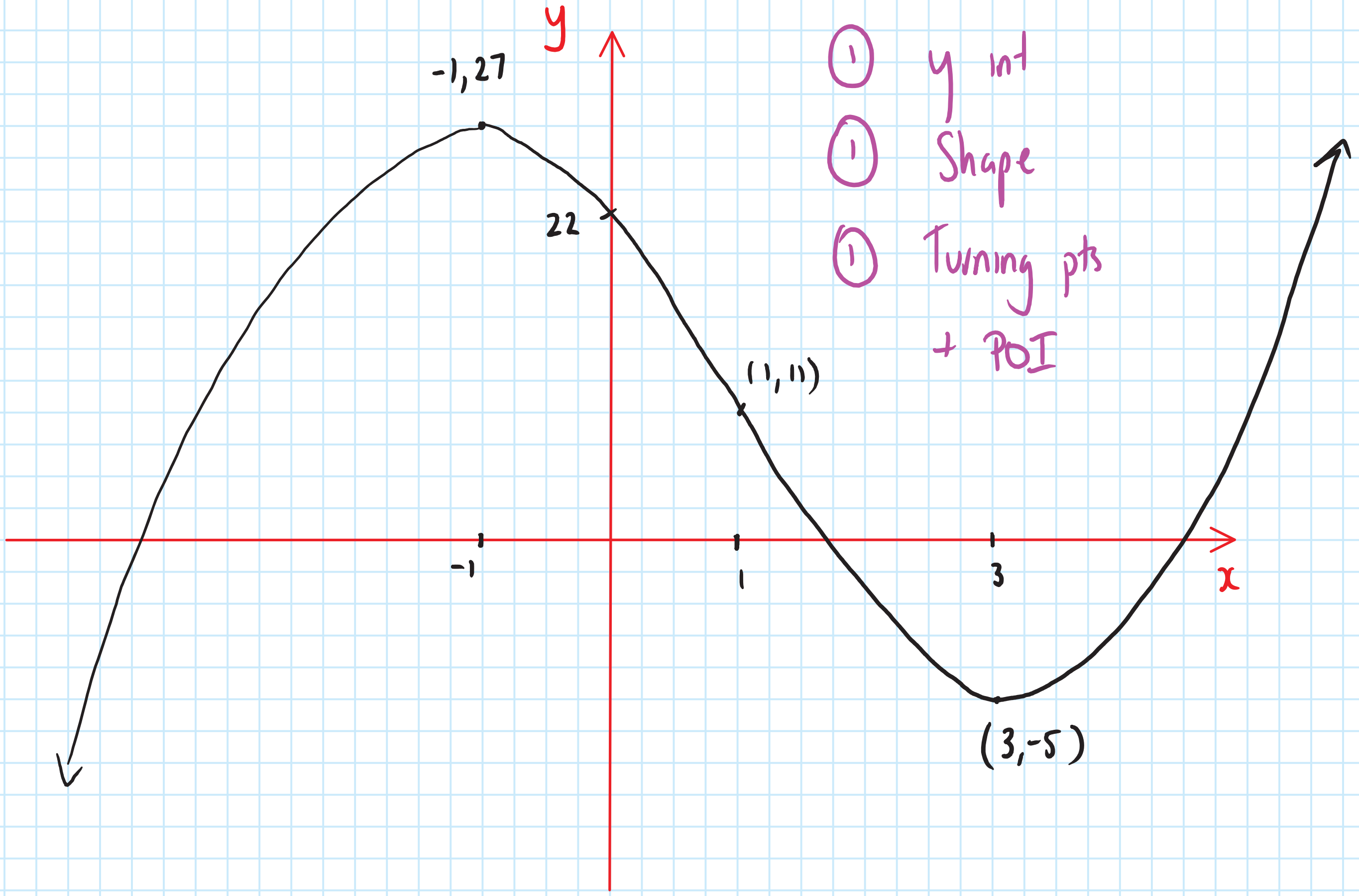
$x = 1$ 1 $y = 11$

x	0.9	1	1.1
$f''(x)$	-0.6	0	0.6

1 TEST

Concavity change

$\therefore$ POI @ $(1, 11)$



(b) (i) $y = 4x^2(1-x)$

$= 4x^2 - 4x^3$

$\frac{dy}{dx} = 8x - 12x^2$

$= 4x(2-3x)$

When $x = 1$ $\frac{dy}{dx} = 4(2-3)$

$= -4$ As required

(ii) Tangent

$$-4 = \frac{y-0}{x-1}$$

$$-4x + 4 = y$$

$$4x + y - 4 = 0 \quad 1$$

Normal

$$\frac{1}{4} = \frac{y-0}{x-1}$$

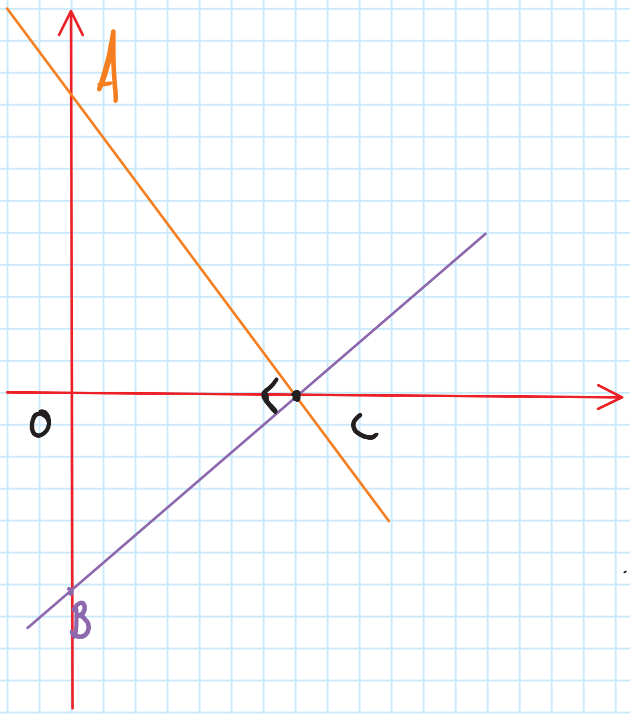
$$x - 1 = 4y$$

$$x - 4y - 1 = 0 \quad 1$$

(iii) A ⇒ y int of tangent

B ⇒ y int of normal

$A(0, 4) \leftarrow$ 1 $\rightarrow B\left(0, -\frac{1}{4}\right)$



As $AC \perp BC$

we could use

$$\text{Area} = \frac{1}{2}(AC \times BC)$$

But $OC \perp AB$ 1

$$\therefore \text{Area} = \frac{1}{2}(AB \times OC)$$

$$= \frac{1}{2}\left(\frac{17}{4} \times 1\right)$$

$$= \frac{17}{8} \text{ units}^2 \quad 1$$

Question 16, Page 1

Wednesday, 12 July 2017 11:12 AM

(a) (i)

$A_0 = 400000$ let $r = \frac{0.06}{12}$

$= 0.005$

$A_1 = 400000 \times (1+r) - M$

$A_2 = A_1 \times (1+r) - M$ 1 Building

$= 400000(1+r)^2 - M(1+r) - M$

~~A_3~~ $A_2 = 399201.61$

$399201.61 = 400000(1+r)^2 - M(1+r+1)$

$$M = \frac{400000(1+r)^2 - 399201.61}{(2+r)}$$

1 Solving for M

$= \$2398.20$ As required.

(ii) $A_3 = A_2 \times (1+r) - M$

$$= 400000(1+r)^{3} - M\underbrace{\left((1+r)^2 + (1+r) + 1\right)}_{\text{3 terms}}$$

SAME One less one less

$$A_n = 400000(1+r)^{n} - M\left((1+r)^{n-1} + (1+r)^{n-2} + \dots + (1+r) + 1\right)$$

SAME

1

$$A_n = 400000(1+r)^n - M\left((1+r)^{n-1} + (1+r)^{n-2} + \dots + (1+r) + 1\right)$$

This is a G.P

$a = 1$

$r = 1+r$

$n = n$

$$A_n = 40000(1+r)^n - M\left[\frac{1\left((1+r)^n - 1\right)}{1+r-1}\right] \quad 1$$

$$= 40000(1+r)^n - M\frac{\left((1+r)^n - 1\right)}{r}$$

Subbing in M & r

$$A_n = 40000(1.005)^n - \frac{2398.20\left((1.005)^n - 1\right)}{0.005}$$

$$= 40000(1.005)^n - 479640(1.005)^n + 479640 \quad 1$$

$$= 479640 - 79640(1.005)^n \quad \text{As required}$$

Wednesday, 12 July 2017 11:12 AM

(iii) $A_n < 150000$

$$479640 - 79640(1.005)^n < 150000$$

$$-79640(1.005)^n < -329640$$

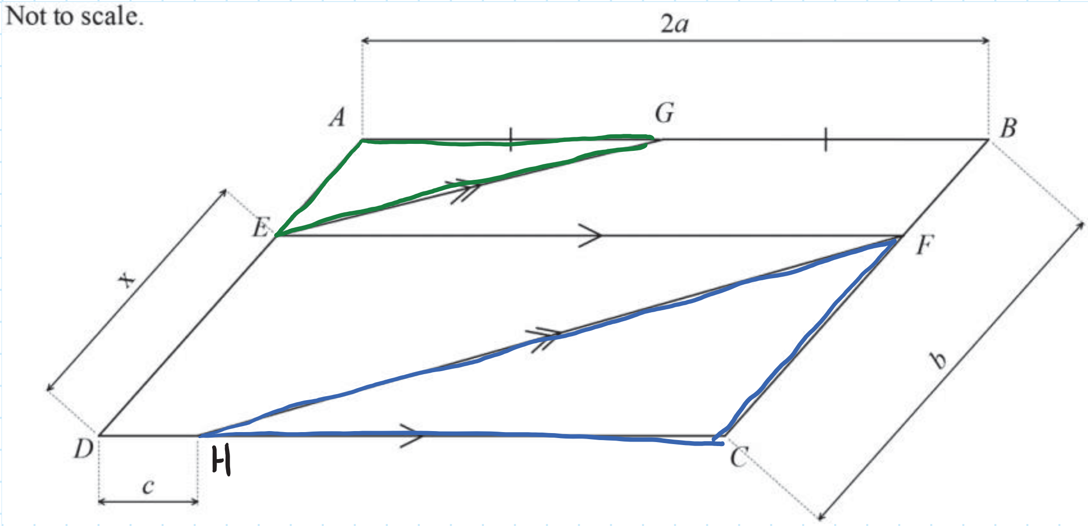
$$(1.005)^n > 4.139126\ldots$$

$$n > \frac{\log 4.139126\ldots}{\log 1.005}$$ 1

$$n > 284.806\ldots$$

$\therefore$ Owing less than \$150000 after 285 months 1

(b)



(i) Prove $\Delta AEG \parallel\!| \Delta CFH$

$\angle EAG = \angle FCH$ (Opposite $\angle$s of a parallelogram are equal)

① 2 angles

let $\angle AGE = \alpha$

$\angle GEF = \alpha$ (Alternate $\angle$s on parallel lines AB & EF are equal)

$\angle EFH = \alpha$ (Alternate $\angle$s on parallel lines EG & HF are equal)

$\angle FHC = \alpha$ (Alternate $\angle$s on parallel lines CD & EF are equal)

$\therefore \Delta AEG \parallel\!| \Delta CFH$ (equiangular) ①

(iii) EDCF is a parallelogram

$\therefore CF = x$ (opposite sides of a parallelogram are equal)

$\therefore AE + x = b$

$AE = b - x$

1 Side length of triangles in a, b, c & x

$AG = a$

$HC = 2a - c$

$$\frac{CF}{HC} = \frac{AE}{AG}$$

(Corresponding sides of similar triangles are in a fixed ratio) 1

$$\frac{x}{2a-c} = \frac{b-x}{a}$$

$$ax = (2a-c)(b-x)$$

$$ax = (2a-c)b - (2a-c)x$$

$$ax + (2a-c)x = (2a-c)b$$

$$(3a-c)x = (2a-c)b$$

$$x = \frac{(2a-c)b}{3a-c}$$

1

(c) $y = b - ax^2$ $\quad a > 0$, $b > 0$ $\quad y = \dfrac{4}{x^2+1}$

2 points of contact

(i) $$b - ax^2 = \frac{4}{x^2+1}$$

$$(x^2+1)(b-ax^2) = 4$$
$$bx^2 - ax^2 + b - ax^4 = 4$$
$$ax^4 + ax^2 - bx^2 + 4 - b = 0$$
$$ax^4 + (a-b)x^2 + 4 - b = 0$$

(ii) This only has one solution for x^2

$$\therefore \Delta = 0 \quad 1$$
$$(a-b)^2 - 4 \times a \times (4-b) = 0$$
$$a^2 - 2ab + b^2 - 16a + 4ab = 0$$
$$a^2 + 2ab + b^2 - 16a = 0$$
$$(a+b)^2 = 16a$$
$$a+b = \pm 4\sqrt{a}$$

Only +ve as $a+b>0$

$$b = 4\sqrt{a} - a$$

As required 1

Question 16, Page 7

Friday, 21 July 2017 12:15 PM

(ii) If there a 2 solutions

then $x^2 = \frac{-(a-b)}{2a}$

$\therefore \quad x^2 > 0$

$\frac{b-a}{2a} > 0$

$2a \longrightarrow$ doesn't matter

1 All steps shown

$(4\sqrt{a} - a) - a > 0$

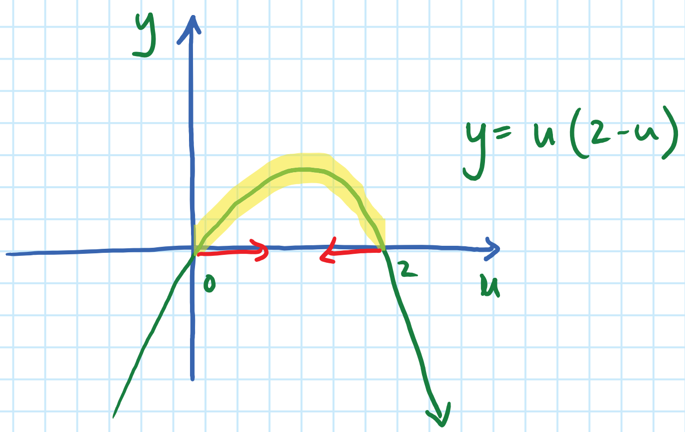
$4\sqrt{a} - 2a > 0$

$2\sqrt{a} - a > 0$

let $u = \sqrt{a}$

$2u - u^2 > 0$

$u(2-u)$



$\therefore \quad 0 < u < 2$

$0 < \sqrt{a} < 2$

$0 < a < 4$

As required